## Natural Frequency for Soil Deposit－NAFR

The program NAFR（Natural Frequency of Soil Deposit）is a subroutine subprogram that looks for peaks in the amplified spectrum in order of decreasing frequency and finds the natural frequencies from the first order to the specified order or to the highest order that can be found．．

## NAFR（Natural Frequency of Soil Deposit）

## 【Purpose】

To compute the natural frequency of the ground from the peak position of the amplified spectrum．

## 【Usage】

（1）How to connect
CALL NAFR（N，DF，FREQ，AMP，ND1，NMODE，F，ND2）

| Argument | Type | Parameter in calling program | Return Parameter |
| :---: | :---: | :---: | :---: |
| N | I | Total number of amplified spectrum | Unchanged |
| DF | R | Frequency increment of amplified spectrum（unit：Hz） | Unchanged |
| FREQ | R 1－D array （ND1） | Frequency string of amplified spectrum （unit ：Hz） | Unchanged |
| AMP | R 1－D array （ND1） | Amplified spectrum data | Unchanged |
| ND1 | I | Dimension size of FREQ and AMP in calling program | Unchanged |
| NMODE | I | No need to input here | Highest order of natural frequency obtained |
| F | R 1－D array （ND2） | No need to input here | Natural Frequencies（unit ： Hz ） |
| ND2 | I | Dimension size of F in calling program | Unchanged |

（2）Necessary subroutines and function subprograms
None
（3）Remarks
It is not able to find natural frequencies above the $N D 2$ order．Therefore，$N D 2$ can be used to specify the highest order of the natural frequency to be determined．

## 【Calculation Method】

The point where the data of the amplification spectrum shows the peak and the two points before and after it are taken as a total of three points，and the frequency of the point where the quadratic curve passing through these three points shows the maximum value is defined as the natural frequency．In other words，if the number of data in the amplified spectrum is $N$ ，and the three successive data $A_{k-1}, A_{k}$ ，and $A_{k+1}$ （ $k=2,3, \cdots, N-1$ ）are

$$
A_{k-1} \leq A_{k} \geq A_{k+1}
$$

then the peak lies between $A_{k-1}$ and $A_{k+1}$ ．Then，when the frequencies of the three points $A_{k-1}, A_{k}$ ，and $A_{k+1}$ are $f_{k-1}=f_{k}-\Delta f, f_{k}, f_{k+1}=f_{k}+\Delta f$ ，respectively，the equation of the quadratic curve passing through the three points is

$$
\begin{equation*}
A=a f^{2}+b f+c \tag{a}
\end{equation*}
$$

where

$$
\begin{gathered}
a=\frac{A_{k-1}-2 A_{k}+A_{k+1}}{2(\Delta f)^{2}} \\
b=-\frac{\left(A_{k-1}-A_{k+1}\right) \Delta f+2 f_{k}\left(A_{k-1}-2 A_{k}+A_{k+1}\right)}{2(\Delta f)^{2}}
\end{gathered}
$$

Therefore，by substituting $a$ and $b$ in the above equation for the value of $f=-b / 2 a$ ，which is obtained from the condition $d A / d f=2 a f+b=0$ in equation（a），the frequency that gives the peak is determined by the following equation．

$$
f=f_{k}+\frac{A_{k-1}-A_{k+1}}{A_{k-1}-2 A_{k}+A_{k+1}} \cdot \frac{\Delta f}{2}
$$

To obtain the amplified spectrum，you can use the subroutine FESP described separately．

## 【Program List】

| C | $* * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ | NAFR 1 |
| :---: | :---: | :---: |
| C | SUBROUTINE FOR Natural frequencies of Layered Soil | NAFR 2 |
| C | ＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊ | NAFR 3 |
| C |  | NAFR 4 |
| C | CODED BY Y．OHSAKI | NAFR 5 |
| C |  | NAFR 6 |
|  | SUBROUTINE NAFR（N，DF，FREQ，AMP，ND1，NMODE，F，ND2） | NAFR 7 |
| C |  | NAFR 8 |
|  | DIMENSION FREQ（ND1），AMP（ND1），F（ND2） | NAFR 9 |
| C |  | NAFR 10 |
|  | NMODE $=0$ | NAFR 11 |
|  | D0 $110 \mathrm{~K}=2, \mathrm{~N}-1$ | NAFR 12 |
|  | IF（NMODE．EQ．ND2）RETURN | NAFR 13 |

```
    IF (AMP (K-1). GT. AMP (K). OR. AMP (K). LT. AMP (K+1)) GO TO 110 NAFR 14
    NMODE=NMODE+1 NAFR 15
    F (NMODE ) = (AMP (K-1) - AMP (K+1)) / (AMP (K-1) -2.*AMP (K) +AMP (K+1)) NAFR 16
    * *DF/2. +FREQ (K) NAFR 17
1 1 0 \text { CONTINUE NAFR 18}
    RETURN NAFR 19
    END NAFR 20
```


## 【Example】

Calculate the amplification spectrum of the ground from the first to the third layer of the four-layer model ground with the characteristic values shown in the table below (the damping coefficients are $\alpha=2.0$ and $\beta$ $=0.02$ for each layer), and calculate the natural frequency and natural period using NAFR based on this spectrum..

| Layer <br> No. | Upper <br> depth <br> $(\mathrm{m})$ | Layer <br> thickness <br> $(\mathrm{m})$ | Unit volume <br> weight <br> $\left(\mathrm{tf} / \mathrm{m}^{3}\right)$ | Elastic shear <br> modulus <br> $\left(\mathrm{tf} / \mathrm{m}^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.00 | 3.80 | 1.50 | 1200 |
| 2 | 3.80 | 3.20 | 1.67 | 2900 |
| 3 | 7.00 | 3.90 | 1.85 | 5700 |
| 4 | 10.90 | - | 1.95 | 50000 |

C
COMPLEX $\quad Z(1000)$
DIMENSION TH (4), UW (4), G (4), ALPHA (4), BETA (4) , FREQ (1000),

* $\quad$ AMP (1000) , F (5)

DATA L/4/, TH/3.8,3.2, 3.9, 0.0/, UW/1.50, 1.67, 1.85, 1.95/,

* G/1200., 2900. ,5700. , 50000/, ALPHA/4*2.0/, BETA $/ 4 * 0.02 /$,
* LOBJ/1/, LREF/4/, N/1000/, DF/0.02/

C
CALL FESP (L, TH, UW, G, ALPHA, BETA, 4, LOBJ, LREF, DF, N, Z, 1000, 0, 0)
D0 $110 \mathrm{~K}=1$, N
$\operatorname{FREQ}(\mathrm{K})=\operatorname{REAL}(\mathrm{K}-1) * \mathrm{DF}$
$\operatorname{AMP}(\mathrm{K})=\operatorname{CABS}(\mathrm{Z}(\mathrm{K}))$
110 CONTINUE
CALL NAFR (N, DF, FREQ, AMP, 1000, NMODE, F, 5)
WRITE $(6,601)$ LOBJ, LREF
DO $120 \mathrm{~J}=1$, NMODE
$\mathrm{T}=1.0 / \mathrm{F}(\mathrm{J})$
WRITE (6, 602) J, F (J), T
120 CONTINUE
STOP
C

```
601 FORMAT (' LAYER', I2' / LAYER', I2//T6, 'MODE', TR4,' FREQ. (HZ)',
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    * TR3,' PERIOD (SEC)’ )
    602 FORMAT (T6, I3, F13. 3, F12. 3)
        END
    
## Output:

LAYER 1/ LAYER 4

| MODE | FREQ. (HZ) | PERIOD (SEC) |
| :---: | :---: | :---: |
| 1 | 3.533 | 0.283 |
| 2 | 8.219 | 0.122 |
| 3 | 14.037 | 0.071 |
| 4 | 18.849 | 0.053 |

